Strategy representation complexity in an evolutionary n-Players Prisoner's Dilemma model

Inácio Guerberoff Lanari Bó[1], Jaime Simão Sichman[1]

[1]Laboratório de Técnicas Inteligentes – Escola Politécnica
Universidade de São Paulo (USP)
Av. Prof. Luciano Gualberto, trav.3, n.158 – Cidade Universitária
05508-900 – São Paulo – SP – Brazil

{inacio.bo,jaime.sichman}@poli.usp.br

Abstract. *This work presents the results of experiments made with a spatial evolutionary model of agents playing the n-Players Prisoner's Dilemma, using two different ways to represent the agent's strategies: finite automata and adaptive automata. Since adaptive automata can represent complex strategies that cannot be represented by finite automata, comparative analysis of the co-evolution of strategies using both representations may lead to a better understanding of the role of complex strategies in evolutionary games. Here are presented the differences observed on the total utility obtained by the agents, the speed in which they converge to a nearly-stationary state, and the characteristics of the prevailing strategies.*

1. Introduction

The Prisoner's Dilemma[Poundstone 1993] is widely used as a paradigm for the study of the evolution of cooperation in a society of agents. It's adequacy for this task comes from the fact that it represents in a very simple way the dilemma faced by agents who may choose or not to cooperate with each other in a non-zero-sum game[1]. It can be described as follows:

> Two suspects of a crime are arrested by the police and interrogated simultaneously and at different rooms. Each one has two choices: remain silent or to accuse the other suspect. If one suspect accuses the other suspect and he remains silent, the one who accuses goes free, while the other receives a 10-year sentence. If both remain in silence, both spend six months in jail, and finally if one accuses the other, both receive a 5-year sentence.

It is clear that the best choice for both agents is to remain in silence, assuring that each one will spend only six months in jail. However, if we look at the decision of each agent, we see that these choices will probably not happen. From the perspective of a single agent in the game, it has only two choices: accuse (which we'll call "defect") and remain in silence (which we'll call "cooperate"). If he defects, the possible outcomes for it are to spend 5 years in jail (if the other player defects

[1]A game where the utility obtained by an agent is not exactly balanced by the utility lost by the other agent. This means that the sum of the utilities earned depends on the choices made by them.

too) or to go free (if the other agent cooperates). On the other hand, if the agent cooperates, he has two other possible outcomes: spend six months in jail (if the other agent also cooperates), or 10 years (if the other agent defects). Since the agent doesn't have how to know what the other will do, the safest choice for him, no matter what the other player does, is to defect.

If we assume that both agents are rational and have the same description of the rules, the result of the game is that both defect, so they both spend five years in jail. This happens even though they could achieve a much better result if both cooperated. No cooperation is achieved because the lack of knowledge on what the other agent will do makes the expected outcome from defecting greater than cooperating. If both could make some kind of agreement, the results would be different.

In essence, the dilemma is that cooperating is good only if the other also cooperates. This situation is seen in many social and biological situations: traffic behavior, cartels in economic markets, arms race between countries, etc.

We have seen that if both agents are rational and have the same description of the rules, no cooperation is achieved in one round of the game. However, if both players play for more than one round, the situation changes. This is called the *Iterated Prisoner's Dilemma*[Axelrod 1997b]. Formally, in this version, with two participants, each one opts simultaneously each round to one between two options: to cooperate ("C") or to defect ("D"). The players get a utility R in case both play "C", and P in case both plays "D". On the other hand, in case a player chooses "C" and the other "D", the player who cooperated receives S and the one who defected receives T, where $T > R > P > S$ and $R > 1/2(T + S)$. This means that an agent has a great incentive to defect (with the perspective of earning T). However, the global result is bigger when both cooperate (because $2R > T + S$), and in fact the only Nash equilibrium[2] for the game is the one where both play "D". If the game will be repeated indefinitely, however, there is the possibility for the agents to establish mechanisms that support a mutual cooperation between them, because the lack of knowledge on the number of interactions that will occur between them makes advantageous the mutual cooperation. It is fundamental, however, that the strategies of the players have a way to deal with free-riders, through some mechanism of punishment.

Since the experiment of competition of strategies for the Iterated Prisoner's Dilemma made by Axelrod [Axelrod 1997b], many works have been made starting from the same principle - agents with heterogeneous strategies playing between them in an evolutionary environment - to analyze different configurations of games, types of strategies and other variables.

[Delahaye and Mathieu 1994] had carried through an analogous competition to Axelrod's in a modified version of the Prisoner's Dilemma, where the agents could oppose to play with the opponent, and verified that strategies with high degree of complexity presented good results in the long-term. [Ifti et al. 2004] used a con-

[2]The Nash equilibrium is combination of choices in a game where no agent can improve his outcome by changing his choice

tinuous version of the Dilemma where agents distributed in a space had only local interactions, and verified that this restriction can lead to the formation of "cooperative clusters", increasing the ratio of cooperative plays.

Other works don't start with a pre-defined set of strategies, but through genetic and evolutionary mechanisms study the development and evolution of them. [Lindgren and Nordahl 1994] use genetic algorithms and strategies with changeable memory size in a space model where the agents play the Prisoner's Dilemma. [Eriksson and Lindgren 2005] uses finite automata to represent the strategy of agents where the payoff matrices can be randomly modified.

1.1. n-Player Prisoner's Dilemma

The generalization of the Prisoner's Dilemma for more than two players (n-Player Prisoner's Dilemma, or NPPD) presents more complex situations and new challenges for the participant agents. Formally, in the NPPD the utility an agent gets depends on its own play ("C" or "D") and on the amount of other agents who cooperated in the same round. Calling $V(C|i)$ the utility earned by a player who played "C" where i other agents cooperated, and $V(D|i)$ the utility earned by the player who played "D" in the same situation, the following conditions defines the NPPD:

$$V(D|i) > V(D|i-1) \tag{1}$$

$$V(C|i) > V(C|i-1) \tag{2}$$

$$V(D|i) > V(C|i) \tag{3}$$

$$(i+1)V(C|i+1) + (n-i-1)V(D|i+1) > iV(C|i) + (n-i)V(D|i) \tag{4}$$

The inequality 1 and 2 reflect the fact that, independently of the play made by the agent, its utility will be bigger the more agents cooperate in the same round. The inequality 3 says that, for the same number of players cooperating, to play *defect* individually produces a better result than *cooperate*. Finally, inequality 4 demands that, in a group with n participants, in case a player pass from **D** to **C** (therefore, starts to cooperate), the total utility of the group increases.

In [Lindgren and Johansson 2001] are presented the following equations for V, which obeys the restrictions of the definition of the NPPD:

$$V(C|n_C) = \frac{n_C}{n-1} \tag{5}$$

$$V(D|n_C) = \frac{T.n_C}{n-1} + \frac{P(n-n_C-1)}{n-1} \tag{6}$$

The constants T and P reflects, respectively, the advantage of defecting (*temptation score*) and the punishment for the mutual defecting. Is assumed, therefore, $1 < T < 2$ e $0 < P < 1$. Finally, n_C is the total number of cooperating agents.

[Glance and Huberman 1994] shows that NPPD games have some characteristics that turns strategies based on reciprocity - as in [Axelrod 1997b] *tit-for-tat* - inefficient. This occurs because there is no possibility of, through the game, penalizing an agent without affecting the other participants. Even though, the authors show that there are two sufficiently stable equilibria for populations playing the NPPD: one with few agents cooperating and another one with many. The study of the stability of these equilibria shows, also, that when occurs transitions between these equilibria, the change will be very fast.

The role of increasing the number of players and the size of the memory used in the strategy of the agents (that is, how many results of previous plays are considered to decide the next play to be made) is analyzed by [Hauert and Schuster 1998], through many simulations using 2, 3 and 4 participants in each game and different sizes of memory. They observe that the growth in the number of participants makes it difficult to establish cooperation, however regarding the size of the memory, the relation is inverse: a minimum size is necessary so that the cooperation can be established continued.

Finally, [Lindgren and Johansson 2001] develops a model where agents distributed in space participate of NPPDs with five participants. The strategies of the agents are represented by finite automata, and new strategies appear through mutations which occurs during the inheritance phase of the evolutionary model.

This work presents a contribution to the study of the evolution of the cooperation, starting from the model proposed in [Lindgren and Johansson 2001], and introducing the use of adaptive automata for the representation of the agents's strategies. Due to its capacity to represent more complex strategies, the comparative analysis of the evolution of both models can help better understanding the role of the complexity of the strategies in these situations.

2. The Model

2.1. Evolutionary environment

The most important characteristic of the environment created for the simulations here presented is that it is evolutionary. This means that the components of the simulation (the agents) are born, reproduce and die throughout time. If we add to this the fact that the probability of reproduction is a result of some desired characteristic (as performance in some task, for example) and the possibility of mutation in the characteristics of the agent in its reproduction, we have as result a system that reproduces, in essence, the mechanism of natural selection.

Also, the environment used in this work has geographic distribution, limiting the interactions between the agents to its neighbors. When the simulation environment does not uses this restriction, generally an agent can interact with all

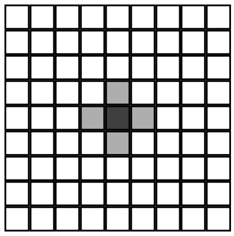


Figure 1. Grid

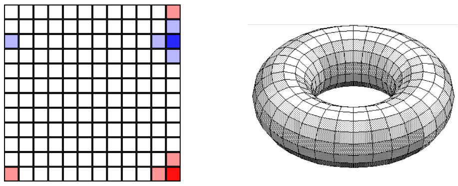


Figure 2. Neighborhood on the edge and torus

the available agents, which can modify the dynamics of the system, and also in its equilibrium results[Lindgren and Nordahl 1994].

The geographic space of this environment is represented by a grid of arbitrary size, whose bidimensional projection can be seen in figure 1. In it, each square is occupied by an agent (if the bidimensional projection has 50 cells in each column and 50 cells in each line, for example, it will have 2500 agents). Each agent has neighbors, following a neighborhood criterion. Here we will use the neighborhood of Von Neumann of degree 1, that considers neighbors the four cells immediately connected to its four sides (as shown in figure 1). The cells in the edges of the grid have neighbors on the opposite sides of the grid. Figure 2 shows two examples of this case. Considering this neighborhood in the edges, the simulation space can be better understood as a discrete *torus* (figure 2).

We have, therefore, that each agent interacts in each round with four neighbors, configuring NPPDs of 5 participants. If we define the grid as being of size mxn, for example, there will be $m.n$ games with 5 participants in each generation. As each game will be repeated, between the same participants, n_{rep} times, there will be $m.n.n_{rep}$ rounds in each generation. Since the strategies are not modified within a generation, the order of choice in which the agents are chosen to play is irrelevant. In other words, the choice of the next agent to interact with its neighbors during the simulation can be sequential or random.

After all the agents have played with their neighbors in a generation, each one will have participated in $5n_{rep}$ games, because he participates in games where it is in the center of the neighborhood as well as in the games where its four neighbors are in the center of the neighborhood. This means that, even though it has only four neighbors, in each generation an agent participates in games with 12 other agents, in groups of 5 (figure 3 shows the player in question in dark ash, the four neighbors with which participate in all $5n_{rep}$ games with horizontal crosshatches, and the eight players with which he participates indirectly through the games of its neighbors with vertical crosshatches).

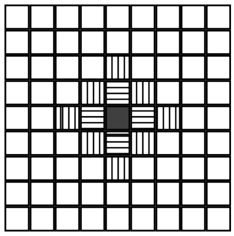


Figure 3. Extended neighborhood

At the end of each generation (when all the agents had completed its series of games), each agent accumulated a utility, resultant of the sum of all the individual results in its $5n_{rep}$ games. This utility reflects the performance that the agent got playing through its strategy. In other words, it indicates the adequacy of the strategy in providing good results for him, and will be criterion used to decide the reproduction of strategies. This way, in the end of a generation each agent compares its total utility with the one of its four neighbors. In case its utility has been bigger or equal to all the utilities earned by its neighbors, it remains with the same strategy. In the other case, he will copy (inherit) the strategy of the agent, in the same neighborhood, that earned the biggest utility. In case more than one has same punctuation, the choice will be random between them. This operation must be made in parallel for all the agents in the end of each generation (so an occurred inheritance is not propagated in the same generation).

During the inheritance a mutation can occur with a probability P_m [3]. For example, if the inheritance occurred without mutation, we would have that an agent **A**, who has the strategy E_a would inherit the strategy E_b from an agent **B**. At the end of this process, **A** would have the strategy E_b and **B** would still have the strategy E_b. However, in case a mutation occurs in this process , **B** will still be with the strategy E_b, but **A** will receive a strategy $E'_b \neq E_b$. The mutations are small changes in the structure that represents the strategy, as the addition or removal of transition or state.

2.2. Strategy representation

2.2.1. Finite Automata

The strategy of an agent must say which is the play that it must do in each round. They can be extremely simple strategies (as "always play C") up to most complex ones (as strategies based of the plays history). In this model the strategies of the agents are represented by finite automata. Automata are structures basically composed by three components:

- A set of states
- A transitions function
- A initial state

Figure 4 shows a strategy for the NPPD with 5 participants. It has three states: **D1**, **C1** and **D2**. The arcs represent the transitions, and the numbers

[3] Where P_m is the same for all agents

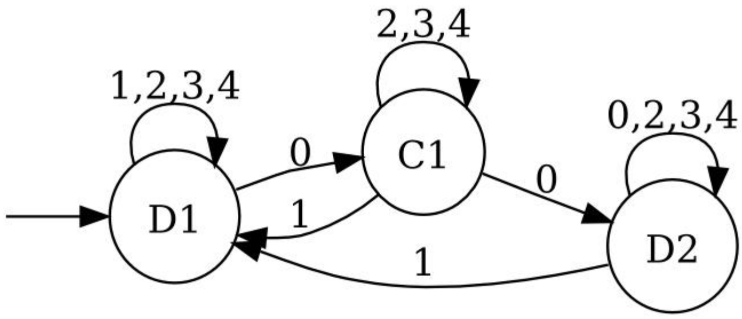


Figure 4. Strategy by finite automaton

associated with them represent the input that triggers the transition. The input is the number of other players who cooperated. The initial state is the one with an empty arrow in its direction (in this in case, it is **D1**). At every moment, the current state defines which will the play of the agent: “C” for cooperate and “D” for defect. The numbers associated to the letters of the states are only used to differentiate them.

The following sequence of plays shows how this strategy may be used:

1. At the first round the agent plays “D” (since his initial state is **D1**)
2. In this round, none of the other four players played cooperate
3. The transition “0” changes the current state from **D1** to **C1**
4. The agent plays “C” (since his current state is **C1**)
5. In this round, two other players cooperate, and the other two plays “D”
6. The transition “2” takes from **C1** to **C1** itself, so the current state is still **C1**
7. In the next round, the agent will play “C”

Each agent has its own strategy, and plays in accordance with it. Strategies represented by finite automata make possible a vast gamma of varieties, however they have limitations. Finite automata are capable to recognize only regular languages in the Chomsky hierarchy[Lewis and Papadimitriou 2004]. This means that they have the guarantee to arrive at a defined state only for a determined class of sequences. In terms of strategies for the NPPD, behaviors as learning and recognition of patterns cannot be represented through them.

2.2.2. Adaptive Automata

Adaptive automata [Neto 2001] are a class of automata that have the ability to modify its own structure, in accordance with the input they receive. These changes are executed through adaptive functions, associated to some transitions of the automaton. These functions can add or remove states and transitions to the automaton while it is used. It can be proven [da Rocha and Neto 2001] that adaptive automata have computational power equivalent to a Turing Machine, which means that, if used for representation of strategies, can present more complex behaviors than the finite automata. As an example of complex behavior, learning mechanisms (that

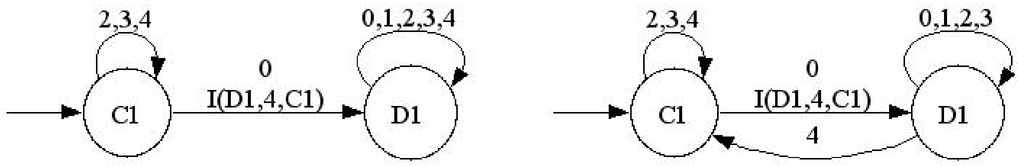


Figure 5. Adaptive automaton before and after the execution of the adaptive function

adapts the behavior of the automaton according to some input pattern) can be built with this structure[Menezes and Neto 2002].

Also, the class of adaptive automata includes the one of finite automata, because when removing adaptive functions of them we get finite automata. This is an important characteristic, that makes possible to use them naturally where the finite automata are used.

In this work, the adaptive functions are always composed by up to three actions of insertion or removal. Figure 5 shows an example. On the left side an adaptive automaton is presented, which has on the transition "0", that goes from **C1** to **D1**, an adaptive function composed by a single action of insertion. Actions of insertion are represented by letter **I** and of removal for letter **R**. The parameters are the origin state, the label of the transition and the destination state). In case the current state is **C1** and the automaton receives input "1", this action is executed, and inserts a transition from **D1** to **C1** for the event "4". Therefore, even though the initial automaton does not have a way to come back to the **C1** state being in **D1**, after the transition from **C1** to **D1** is triggered, this possibility is created.

Beyond referencing existing states, the adaptive actions can have as parameter new states. The action **I(D1,3,C_ref)**, for example, inserts a transition with label "3" that goes from the state **D1** to a new state of type "C". Thus, beyond creating new transitions between existing states, adaptive functions can create new states. It can be proved that, with this specification, it is possible to build strategies that cannot be represented by finite automata.

2.2.3. Mutations

As described in the section 2.1, new strategies appear in the population through mutations, which happens during the inheritance, at the end of a generation. The mutations can be of the following types:

1. Change the initial state of the automaton
2. Change the target of an existing transition
3. Change the type of a state (Ex: from "C" to "D")
4. Add a new state, linking it to an existing one
5. Associate an adaptive function to an existing transition

Mutations 1,2,3 and 4 are present in [Lindgren and Johansson 2001], and are enough to make possible the development of any finite automaton. Mutation 5, on the other hand, is the responsible for more complex strategies. The adaptive functions used in this mutation are generated of randomly, having 1, 2 or 3 actions of insertion or removal.

3. Description of the experiments

To analyze the impact of the use of this new technique on the representation of strategies in the evolution of the cooperation, two experiments had been made: one where the mutation that associates adaptive functions to transitions is allowed and another one where it is not. Consequently, in the last case the strategies are equivalents to those used in [Lindgren and Johansson 2001] and in the other, strategies based on adaptive automata.

The size of the grid used was of 50x50, totalizing 2500 agents, who starts the simulation with the same strategy, composed of only one state of type "D" (that is, they never cooperate). Each simulation was composed by 2700 generations, and, to explore the characteristics of long-term of the strategies, each game in a generation is composed by 150 rounds ($n_{rep} = 150$). In the end of each generation, the automaton is returned to its initial situation (equivalent, in a finite automaton, to return the current state to its initial state and, in the adaptive automaton, to the previous structure before the changes caused by adaptive functions).

In order to represent communication failures and agent's mistakes, in 1% of the rounds the play made by the agents will be the opposite of that defined by its strategy. Also, the utility earned by the agent is deducted by a "complexity cost", proportional to the number of states that composes the strategy. With this, strategies with bigger number of states with equivalent results to those with little states will have lesser probability to be reproduced.

Since the adaptive functions are generated randomly, throughout time its action can become incorrect (for example, indicating the removal of a transition that does not exist anymore). In this case, the function is ignored during the execution.

Finally, the values of T and P of the equation 6 were defined as 1.5 e 0.25, respectively, and the mutation probability (P_m) is defined as 2.5%.

4. Results

4.1. Aggregate data

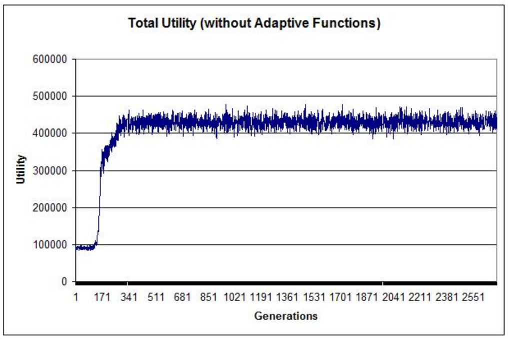
The table 1 shows the aggregate results gotten by the two simulations. It can be observed that, in both cases, there was a fast convergence to a situation of broad cooperation between the agents, even with the incentive to an individualistic and defecting behavior in the definition of the game.

Figures 6(a), 6(b), 6(c) and 6(c) show the total utility earned by the agents throughout time and the total number of plays "C" and "D" in each generation. In them we can see that, after the convergence to a situation of prevalence of cooperation, the system presents a relative stability until the end of the simulation. This means that the strategies of the agents had managed, in both cases, to keep a situation of cooperation with considerable stability.

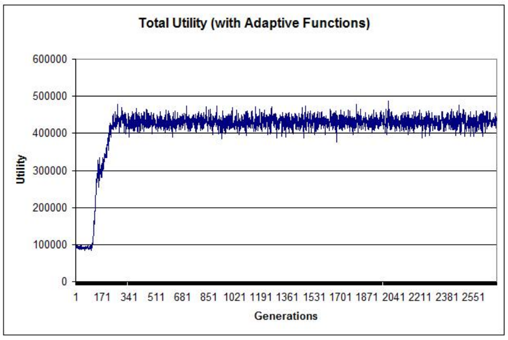
	With adaptive functions	Without adaptive functions
Average utility by generation	413313	409464
Average C plays by generation Standard deviation (σ)	1612881 (86%) 339877	1597642 (85%) 374207
Average D plays by generation Standard deviation (σ)	262051 (14%) 339911	277357 (15%) 374207
Number of generations until C plays prevail	128	157

Table 1. Simulation results

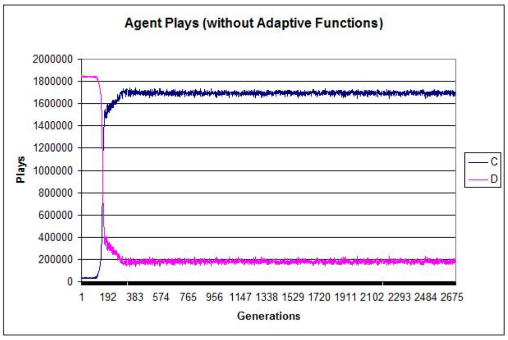
The persistent presence of about 15% of defective plays, however, indicates an equilibrium situation where those plays have an important role in the maintenance of the cooperation [4]. An analysis of the strategies of the agents can help understanding this better.



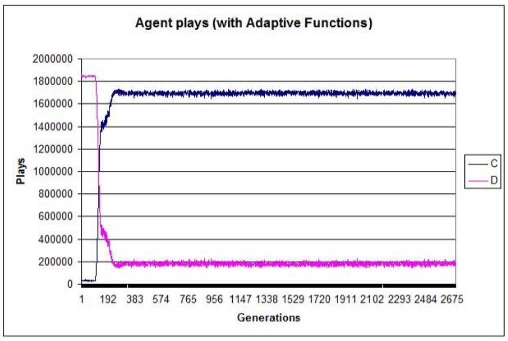
(a) Total Utility without adaptive functions



(b) Total Utility with adaptive functions



(c) Total agent plays without adaptive functions



(d) Total agent plays with adaptive functions

Figure 6. Simulation results

4.2. Analysis of the strategies

Throughout the 2700 generations, 35475 different strategies where developed on the simulation with adaptive functions and 13100 for the simulation without adaptive

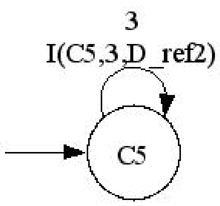
[4]It's important to stand out that random plays caused by agent's mistakes represent only 1% of the plays, not being able, therefore, to explain the observed result

functions. Even though, in the last generation the strategies with bigger population had basically the same principle. Figure 7 shows these strategies, for both cases.

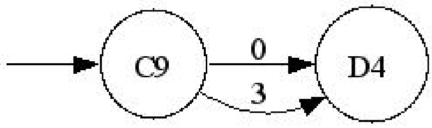
The strategy that prevails at the end of the simulation with adaptive functions (figure 7(a)) has only one state and a transition with label “3”. This transition, however, has an adaptive function that inserts a new state of type “D” on this same transition. As result, its behavior can be described by two rules:

- Start playing “C”, and keep this way until the result of the last round wasn't 3 (only one agent defecting)
- In case exactly one other agent doesn't cooperates, starts defecting until the end of the generation

The strategy that prevails in the simulation without adaptive functions (figure 7(b)) has two states, presenting the same behavior that in the case without them, with a difference: beyond starting defecting when one another agent defects, also does it when no other agent cooperates.



(a) Prevailing strategy for simulation with adaptive functions



(b) Prevailing strategy for simulation without adaptive functions

Figure 7. Prevailing strategies after 2700 generations

Considering also the ratio of cooperative plays observed throughout the simulation (table 1 and figures 6(c) and 6(d)), it can be inferred that the stability should be due to the “revengeful” characteristic of the observed strategies. They are based on not forgiving an agent who defects, even though this have a cost for all the other participants.

A situation where the majority of the agents uses strategies with this characteristic isn't favorable, however, to the development of other more complex ones, due to the fact that they, from the moment there was a defecting play, defects indefinitely, creating a “trap”, where the success of different strategies becomes extremely difficult. The fact that this occurred in both the simulations, however, indicates that, given the initial conditions used and the characteristics of the model, the maintenance of the cooperation is only accomplished through the generations, and not during them.

We see, therefore, that even with the introduction of adaptive functions, the co-evolution of strategies did not present a different result from that where simpler strategies had been used, represented by finite automata.

5. Conclusion and future works

This work presented a model for the study of the co-evolution of strategies for the NPPD with five participants, where the strategies used by the agents are developed and selected through a mechanism similar to the natural selection. Two forms of representation of strategies had been tested: finite automata and adaptive automata, being the last one capable of representing more complex strategies.

The comparison of the results obtained in both simulations showed that there was a fast convergence for a situation of broad cooperation between the agents, obtained in both cases through strategies of "revengeful" behavior: the cooperation is kept while all the participants cooperate. From the moment where this does not occur, it will not cooperate until the end of the generation.

It hasn't been observed, however, significant differences in the aggregate data or in the analysis of the dominant strategies on both simulations. A possible explanation for this is the fact that strategies with this "revengeful" characteristic appear quickly through few mutations, and creates a "trap" that hinders the development of different ones.

Future works include improvements on the algorithm of generation of adaptive functions, that today consists of random choices, resulting in a great number of useless functions, thus diminishing the space of possible functions (today about 32000). Simulations with a bigger number of generations will also be made, testing the stability with better precision. Tests with different configurations of initial strategies will also be made, in order to observe the impact of the initial conditions in the development of the system.

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